$^{171}Yb^+$ Microwave Frequency Standard

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***Abstract*— Microwave frequency standards based on the 12.6 GHz ground state hyperfine transition in $^{171}Yb^+$ have been under development at the National Measurement Institute, Australia, for many years. Using a laser-cooled ion cloud, the transition frequency was measured in 2001 to an accuracy of 8 parts in 10^{14}, limited by the homogeneity of the magnetic field. Uncertainties associated with field inhomogeneity in the new vacuum system are now below 1 part in 10^{15}. We demonstrate that other systematic uncertainties such as AC Zeeman shift, microwave imperfections and pressure shifts permit operation in the 10^{-15} accuracy range. The performance of the $^{171}Yb^+$ microwave standard can therefore be comparable to that of a caesium fountain.**

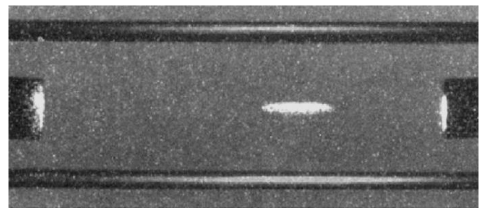


Fig. 1. A laser-cooled $^{171}Yb^+$ ion cloud, approximately 10 mm long with a radius of 1.5 mm. Linear Paul trap electrodes are visible top and bottom (RF, diameter 2.3 mm, separation 20 mm) and left and right (DC, separation 60 mm).

I. Introduction

Microwave frequency standards based on the 12.6 GHz ground-state hyperfine (clock) transition in trapped $^{171}Yb^+$ ions have demonstrated a frequency uncertainty of ±1.1 parts in 10^{13} when operated with a He buffer gas for ion cooling [1], [2]. The greatest contribution to the frequency uncertainty is the second-order Doppler shift due to the ion thermal motion, which can be reduced by eliminating the buffer gas and laser cooling the ions to sub-Kelvin temperatures [3]–[5].

We have previously reported microwave spectroscopy of laser-cooled $^{171}Yb^+$ [4], completed an extensive series of measurements of the ion temperature to quantify the remaining second-order Doppler shifts [5] and made a preliminary measurement of the clock transition frequency using a laser-cooled ion cloud [3]. The limiting uncertainty for this measurement was due to residual magnetic field inhomogeneity associated with the stainless steel ultra-high vacuum (UHV) chamber. We have designed and commissioned a new chamber in the novel alloy CrCu, which is non-magnetic and has good vacuum properties [6]. Recent work has concentrated on completing analysis of systematic shifts in the clock transition frequency, including AC Zeeman shifts, shifts due to imperfections in the microwave spectrum and pressure shifts due to residual background gases.

II. The Microwave Frequency Standard

A. Experimental Apparatus

The trapped-ion frequency standards at the National Measurement Institute, Australia (NMIA) have been described previously [1]. A linear Paul trap (Fig. 1) operates inside a UHV chamber at a base pressure below 1×10^{-10} Torr. The chamber is surrounded by four layers of magnetic shielding, and the ambient magnetic field inside the shields is controlled using three-axis Helmholtz coils plus additional coils to compensate for residual gradients.

The Yb^+ resonance transition near 369 nm (Fig. 2) is used both for laser cooling, and to prepare and probe the populations of the ground state levels; this light is generated by a frequency-doubled titanium-doped sapphire laser. A small fraction of the ions decay to the metastable $^2D_{3/2}$ level; the transition at 935 nm returns these ions to the cooling cycle, using light generated by an external-cavity diode laser. Finally, the transition between the ground state hyperfine levels is the reference frequency for the standard. Microwave radiation near 12 642 812 120 Hz for spectroscopy of this transition is synthesized from a sapphire-loaded superconducting dielectric resonator oscillator [7]. This radiation is also applied during cooling, in order to drain population accumulating in the lower hyperfine level.

Following loading, the RF and DC potentials of the linear trap are reduced for laser cooling. The fluorescence decreases sharply as the cloud approaches its coldest temperature, which we interpret as a 'phase transition' to a liquid-like rather than a crystalline state [8]: we calculate a Coulomb coupling parameter (the ratio of electrostatic potential energy to thermal energy k_BT) of $\eta \sim 0.5$ at $T = 0.4$ K, well below the transition value observed for large ion crystals [9].

B. Measurement of the Transition Frequency

Spectroscopy of the $M_F = 0 \rightarrow 0$ clock hyperfine component of the 12.6 GHz transition uses Ramsey's method, with two $\pi/2$ pulses of length $\tau = 400$ ms separated by $T_R = 10$ s between pulse centres. The 369 nm light is blocked during the interrogation sequence to prevent light shifts, then reapplied to record the ion fluorescence as a measure of the microwave absorption. The ion cloud is subsequently re-cooled for 2 s, with the microwave field switched off

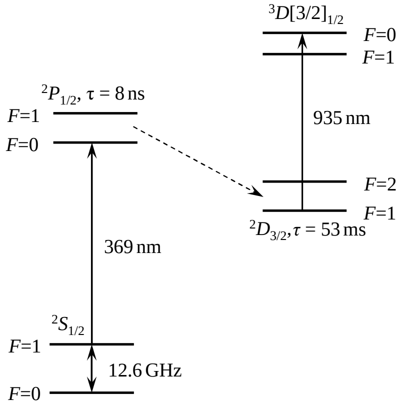


Fig. 2. Partial energy level diagram of the $^{171}Yb^+$ ion.

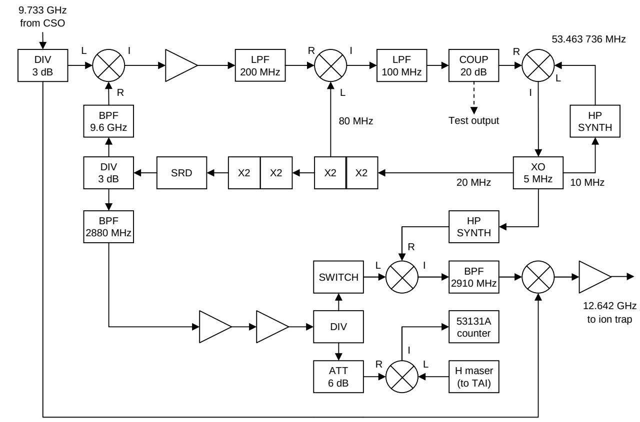
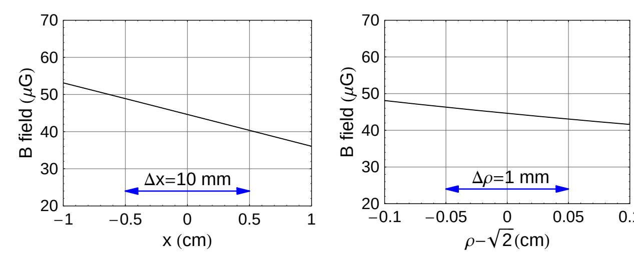


Fig. 3. Block diagram of the frequency synthesis system used to generate the microwave interrogation signals for the ion trap.



(a) (b)

Fig. 4. Theoretical B field distribution generated by the current in one electrode, along the longitudinal direction (a) and the radial direction (b).

for the last part of the cooling to optically pump the ions into the lower hyperfine state for the next interrogation. The microwave frequency is adjusted to match the ion transition by recording fluorescence at the half-height points of the central Ramsey fringe. The ambient magnetic field is determined from the Larmor frequency ν, obtained by similarly tracking the $M_F = 0 \rightarrow \pm 1$ Zeeman component using Ramsey pulses with $\tau = 1$ ms and $T_R = 2$ ms. A measurement cycle consists of a frequency measurement (two count periods) for both the clock and Zeeman components, with a cycle time of approximately 40 s.

The absolute frequency calibration of the applied microwave field is conducted in two stages. A 2.88 GHz frequency signal derived from the sapphire resonator is directly compared to a similar signal from a hydrogen maser (see Fig. 3), which must then be separately compared to the SI second. Since the only local caesium references are commercial beam standards with insufficient frequency accuracy, the calibration must be made by long-range time and frequency transfer. The uncertainty for common-view GPS frequency transfer (5 parts in 10^{14}) was sufficient for the preliminary measurement [3], but more precise methods will be required once the systematic uncertainty in the frequency measurements reaches projected levels.

III. Systematic Shifts

A. AC Zeeman Shift

An oscillating B field is generated near the electrodes by the RF trap voltage and causes an AC Zeeman shift of the clock transition. In principle, this shift can be cancelled due to the symmetric geometry, since currents at frequency Ω in the electrodes of an ideal trap are symmetric with respect to the trap nodal line. However, in practice the AC Zeeman shift can be a significant source of systematic uncertainty [10]. In order to evaluate the possible shift in our system, we consider the shift due to a single electrode, which acts like a monopole antenna with high impedance ($Z_A \approx 1.5 \times 10^{-5} - j2.1 \times 10^5\ \Omega$). The resistance (Re$[Z_A]$) is associated with power loss in the form of antenna radiation and ohmic losses. The reactance (Im$[Z_A]$) is associated with energy storage in the near field of the antenna. The field amplitude B can be obtained from general antenna theory and Ampere's law.

The B field distribution as a function of ρ (radial) and x (longitudinal) is shown in Fig. 4. B at $\rho = \sqrt{2}$ cm, which is the distance between an electrode and the trap center, is 44.6 μG, and the fractional shift is 4.9×10^{-17}. When we consider the size of the ion cloud, 10 mm long with a radius of 1 mm, the variation in B with position is under 10 μG which corresponds to an uncertainty of only 2×10^{-17}. Note that the B field distribution along the longitudinal direction is more significant than in the radial direction because the ion cloud is long and the current distribution on the electrode is linear, not uniform.

Actual shifts are difficult to calculate in practice as apparatus imperfections such as patch charge effects and electrode misalignment are difficult to characterize. However, the shift observed for a linear Hg^+ trap at NIST is of order 10^{-15} [10]; we calculate that shifts for Yb^+ with our trap geometry are smaller by a factor of approximately 500 than the equivalent shift for Hg^+ in the NIST trap, as the trap frequency is lower and the trap dimensions are larger. Allowing for uncertainty in the cloud dimensions and trap geometry, therefore, the AC Zeeman shift for our Yb^+ trap is estimated to be less than 1×10^{-15}.

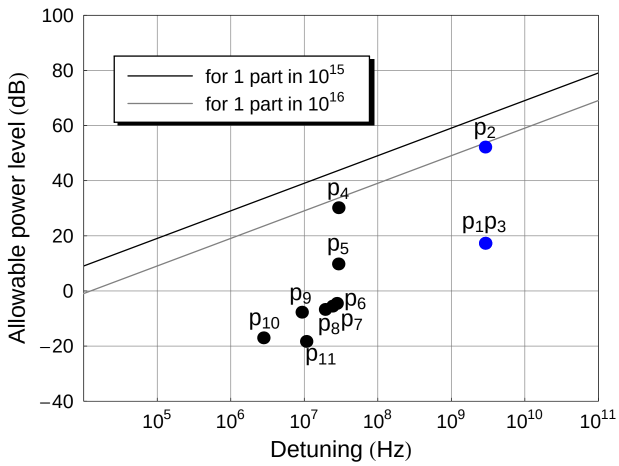


Fig. 5. Fractional frequency shifts ($\Delta f/f_0$) due to the off-resonant components in the microwave spectrum for the Ramsey interrogation pulses.

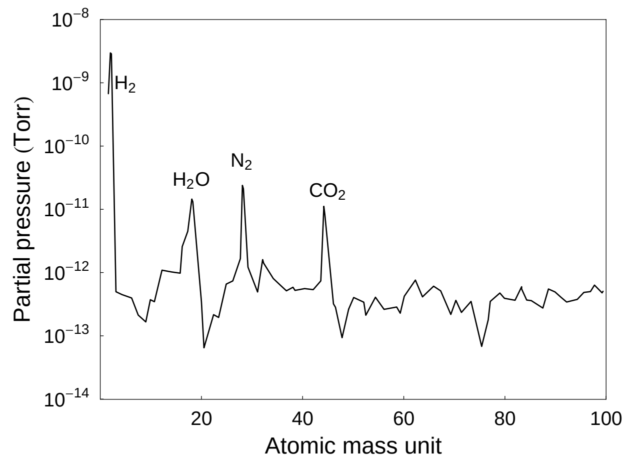


Fig. 6. Residual gas distribution in the trap chamber at UHV.

TABLE I

FREQUENCY SHIFTS DUE TO OFF-RESONANT MICROWAVE COMPONENTS

	detuning (MHz)	relative power (dB)	shift ($\Delta f/f_0$)
p_1	-2910.7	17.3	2.3×10^{-20}
p_2	-2909.3	52.2	7.0×10^{-17}
p_3	-2907.9	17.3	2.3×10^{-20}
p_4	-29.3	30.2	4.4×10^{-17}
p_5	-29.2	9.8	4.1×10^{-19}
p_6	-27.8	-4.5	1.6×10^{-20}
p_7	-24.3	-6.5	1.1×10^{-20}
p_8	-19.3	-6.7	1.4×10^{-20}
p_9	-9.3	-7.7	2.3×10^{-20}
p_{10}	-2.8	-17.0	8.8×10^{-21}
p_{11}	10.7	-18.3	1.7×10^{-21}

TABLE II

PRESSURE SHIFTS KNOWN FOR MOST COMMON GASES

gas	$(\partial\nu/\partial\mathrm{P})/f_0$ (Pa^{-1})	pressure (Torr)	shift ($\Delta f/f_0$)
H_2	1.4×10^{-9}	3.0×10^{-9}	5.6×10^{-16}
He	2.2×10^{-10}	4.5×10^{-13}	1.3×10^{-20}
CH_4	-1.2×10^{-8}	2.6×10^{-12}	-4.2×10^{-18}
Ne	5.3×10^{-10}	3.4×10^{-13}	2.4×10^{-20}

B. Microwave Imperfections

The Yb^+ transition frequency of 12.642 GHz is obtained from a sapphire resonator at 9.733 GHz (Fig. 3). Off-resonant components in the microwave spectrum generated in the resonator lock and synthesis chain [11] cause a shift of the clock transition frequency.

The effect of spurious spectral components or 'spurs' in the spectrum of the microwave excitation in atomic fountains has been previously investigated [12], [13]. The frequency bias due to spurs can appear in any pulsed atomic standard incorporating Ramsey spectroscopy, including trapped ion standards. Here we consider the effect of far off-resonant components, $\Delta \gg 1/\tau$, at $b_0\tau = \pi/2$, where Δ is the detuning of the off-resonant component, b_0 is the Rabi frequency of the resonant component, and τ is the $\pi/2$ pulse duration. In this case the shift varies as $(b_1^2/\Delta)(\tau/T_R)$ [14], where b_1 is the Rabi frequency of the off-resonant component, and T_R is the Ramsey time between excitations. The (τ/T_R) factor is equivalent to the ratio of the Rabi-to-Ramsey lengths (l/L) for a beam standard. We have the following relation between the relative power level of the off-resonant component and the shift (Δf)

$$10\log\left[\frac{P_1}{P_0}\right] = 10\log\left[\left(\frac{T_R}{\tau}\right)\frac{\delta\Delta f}{b_0^2}\right], \qquad (1)$$

where P_0 and P_1 are the powers of the resonant and off-resonant components, respectively, when $b_0\tau = \pi/2$ for the resonant component.

The measured relative power of dominant off-resonant components and the corresponding calculated shifts are shown in Fig. 5 and listed in TABLE I. At present the 9.73 GHz components p_1-p_3 are not switched (Fig. 3), so the shifts by these components are actually (T_R/τ) larger; switching is currently being redesigned to rectify this. Once the switching has been corrected the total shift is below 1×10^{-15}.

Phase shifts during switching in the Ramsey pulse sequence can also cause a systematic frequency shift [1]; this issue is under investigation.

C. Pressure Shift

Pressure measurements for background gases have been made with a residual gas analyzer (RGA) as shown in Fig. 6. The total background pressure of about 5×10^{-10} Torr is mainly due to H_2, H_2O, N_2 and CO_2; this pressure is below 2×10^{-10} Torr when the RGA is off. Fractional pressure shift sensitivities of the 12.6 GHz $^{171}Yb^+$ clock transition have previously been measured for several gases [11], [15]. The magnitudes of pressure shifts for H_2, He, Ne, and CH_4

TABLE III

FRACTIONAL FREQUENCY UNCERTAINTY (10^{-15})

Systematic shift	2001	Present
Second-order Zeeman (DC)	30	< 1
Second-order Zeeman (AC, Ω)	2	< 1
Magnetic imhomogeneity	40	< 1
Second-order Doppler (micromotion)	1	< 2
Second-order Doppler (secular)	0.5	< 1
Quadratic Stark	0.05	0.05
Blackbody radiation		< 0.1
Microwave imperfections	3	< 1
Pressure shift		< 0.6
Combined	~ 60	< 4

are shown in TABLE II. The largest shift is that due to H_2, a fractional shift of 5.6×10^{-16}. The largest pressure shift coefficient is for CH_4. The RGA confirms that the partial pressure for CH_4 is very low; the fractional shift is only -4.2×10^{-18} and reduces the total pressure shift due to the sign of the coefficient. The shift due to N_2 is not known. If ($\partial\nu/\partial$P) was of the same order as for CH_4, the fractional shift would still be below 1×10^{-16} and therefore negligible. For the laser-cooled case, therefore, the pressure shift is less than 1×10^{-15}.

IV. CONCLUSION

The maximum achievable stability for a frequency standard based on laser-cooled $^{171}\mathrm{Yb}^+$ ions is predicted to be better than $\sigma_y(\tau) = 5 \times 10^{-14}\ \tau^{-1/2}$, comparable to that already demonstrated at NMIA for buffer-gas cooled standards. This prediction assumes a cloud of radius 1 mm and length 10 mm containing approximately 10^4 ions, a Ramsey pulse separation of 10 s, and a cycle time of 13 s after allowing for cooling periods. Systematic shifts associated with oscillating B fields due to currents in the trap electrodes, microwave imperfections (with the exception of residual phase shifts) and background residual gas have been characterised to an uncertainty well below 1×10^{-15}. Now that uncertainties associated with magnetic field inhomogeneity have been substantially reduced [6], we see no serious obstacle to realising a projected uncertainty of 4×10^{-15} or lower (see TABLE III). Based on these estimates, a cloud of laser-cooled $^{171}\mathrm{Yb}^+$ ions in a linear trap is projected to exhibit comparable performance to a caesium fountain, and continues to show promise as a frequency standard of both high accuracy and high stability.

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